Roll No. | | | | | | | |

(Write Roll Number from left side exactly as in the Admit Card)

Signature of Invigilators

1. ____________________

2. ____________________

1617

Question Booklet Series **X**

PAPER–II

Question Booklet No.

(Identical with OMR Answer Sheet Number)

Subject Code : 16

PHYSICAL SCIENCES

Time : 1 Hour 15 Minutes *Maximum Marks: 100*

Instructions for the Candidates

1. Write your Roll Number in the space provided on the top of this page as well as on the OMR Sheet provided.
2. At the commencement of the examination, the question booklet will be given to you. In the first 5 minutes, you are requested to open the booklet and verify it:
   (i) To have access to the Question Booklet, tear off the paper seal on the edge of this cover page.
   (ii) Faulty booklet, if detected, should be get replaced immediately by a correct booklet from the invigilator within the period of 5 minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time will be given.
   (iii) Verify whether the Question Booklet No. is identical with OMR Answer Sheet No.; if not, the full set to be replaced.
   (iv) After this verification is over, the Question Booklet Series and Question Booklet Number should be entered on the OMR Sheet.
3. This paper consists of fifty (50) multiple-choice type questions. All the questions are compulsory. Each question carries *two* marks.
4. Each Question has four alternative responses marked: (A) (B) (C) (D). You have to darken the circle as indicated below on the correct response against each question.

   Example: (A) (B) ● (D), where (C) is the correct response.
5. Your responses to the questions are to be indicated correctly in the OMR Sheet. If you mark your response at any place other than in the circle in the OMR Sheet, it will not be evaluated.
6. Rough work is to be done at the end of this booklet.
7. If you write your Name, Roll Number, Phone Number or put any mark on any part of the OMR Sheet, except the space allotted for the relevant entries, which may disclose your identity, or use abusive language or employ any other unfair means, such as change of response by scratching or using white fluid, you will render yourself liable to disqualification.
8. Do not tamper or fold the OMR Sheet in any way. If you do so, your OMR Sheet will not be evaluated.
9. You have to return the Original OMR Sheet to the invigilator at the end of the examination compulsorily and must not carry it with you outside the Examination Hall. You are, however, allowed to carry question booklet and duplicate copy of OMR Sheet after completion of examination.
10. **Use only Black Ball point pen.**
11. **Use of any calculator or mobile phone etc. is strictly prohibited.**
12. **There are no negative marks for incorrect answers.**

[Please Turn Over]

PHYSICAL SCIENCES

Paper II

1. The function $y = x \log(x)$ has

(A) zeroes at $x = 0$ and $x = 1$ and a minimum in between.

(B) zeroes at $x = 0$ and $x = 1$ and a maximum in between.

(C) a singularity at $x = 0$ and increases monotonically for $x > 0$, faster than $y = x$.

(D) a singularity at $x = 0$ and increases monotonically for $x > 0$, slower than $y = x$.

2. Let $\vec{A} = yz\hat{i} + xy\hat{j} + zx\hat{k}$ where $\hat{i}, \hat{j}, \hat{k}$ are three unit vectors along the three rectangular cartesian axes. For a circle of radius R lying on the x - y plane with the centre at the origin, $\oint \vec{A}.d\vec{r}$ [where the integral is to be taken over the circumference of the circle in an anticlockwise direction] is

(A) $\dfrac{R^3}{3}$

(B) $\dfrac{2R^3}{3}$

(C) R^3

(D) 0

3. The dimension of $\delta^3(\vec{p})$, where $\vec{p}$ is the momentum is

(A) $M^3 L^3 T^{-3}$

(B) $M^3 L^{-3} T^3$

(C) $M^{-3} L^{-3} T^3$

(D) No dimension

4. If $f(x) = \begin{cases} x, & -\frac{\pi}{2} < x < \frac{\pi}{2} \\ \pi - x, & \frac{\pi}{2} < x < \frac{3\pi}{2} \end{cases}$ then the coefficient a_0 in the Fourier expansion of $f(x)$ is

(A) 2π

(B) 0

(C) -2π

(D) 1

5. The eigenvalues of a real, symmetric matrix M are $2, 2+\sqrt{2}$ and $2-\sqrt{2}$.

The corresponding eigenvectors (normalised) are

$$\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ \sqrt{2} \\ 1 \end{pmatrix} \text{ and } \begin{pmatrix} 1 \\ -\sqrt{2} \\ 1 \end{pmatrix}$$

respectively. The matrix M is

(A) $\begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix}$

(B) $\begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 0 \\ 1 & 0 & 2 \end{pmatrix}$

(C) $\begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$

(D) $\begin{pmatrix} 1 & -\sqrt{2} & 0 \\ -\sqrt{2} & 2 & \sqrt{2} \\ 0 & \sqrt{2} & 2 \end{pmatrix}$

6. The orthonormal set of vectors ϕ_1, ϕ_2 and ϕ_3 form a basis in a linear vector space. An operator *A* transforms ϕ_1 to ϕ_2, ϕ_2 to ϕ_3 and ϕ_3 to ϕ_1. The matrix representation of *A* in the above basis is given by,

(A) $\begin{pmatrix} 0 & i & -i \\ -i & 0 & i \\ 0 & i & 0 \end{pmatrix}$

(B) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(C) $\begin{pmatrix} 1 & 0 & -1 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \end{pmatrix}$

(D) $\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

7. $r^m e^{in\theta}$, where *m* is a positive integer, is an analytic function in the complex z-plane if

(A) $m = n$

(B) $m = -n$

(C) $m = \pm n$

(D) $n = 0$

8. If $\int_{-\infty}^{+\infty} k(x)\, f^*(x)\, g(x)\, dx$ is to be taken as an inner product of any two functions *f*(*x*) and *g*(*x*) in the space of square integrable complex functions of real variable *x* then a valid choice of *k*(*x*) is

(A) x^3

(B) $e^{\frac{x^2}{2}}$

(C) $e^{-\frac{x^2}{2}}$

(D) e^{ix}

9. The Earth executes bounded motion round the Sun if the total energy *E* of the Earth

(A) $E > 0$

(B) $E < 0$

(C) $E = 0$

(D) $E >> 0$

10. Two particles of equal mass have velocities $\vec{v}_1 = 2\hat{i}\ m/s$ and $\vec{v}_2 = 2\hat{j}\ m/s$. First particle has an acceleration $\vec{a}_1 = \left(3\hat{i} + 3\hat{j}\right) m/s^2$, while the second particle moves without any acceleration. The centre of mass of the system of two particles moves on

(A) a circle

(B) a parabola

(C) a straight line

(D) an ellipse

11. A block of mass *M* is floating in a liquid of density ρ being partially immersed. The block is pushed slightly into the liquid and then released. The time period of its harmonic oscillation obeys the relationship

(A) $T \propto \sqrt{\frac{M}{\rho}}$

(B) $T \propto \sqrt{\frac{\rho}{M}}$

(C) $T \propto \sqrt{\rho M}$

(D) $T \propto \frac{M}{\rho}$

12. Consider the Lagrangian of a classical simple harmonic oscillator. Which one of the following terms, when added to the Lagrangian, will not change the equation of motion? (k is a constant).

(A) $x + k$

(B) $\dot{x} + k\,x$

(C) $\dot{x} + k$

(D) $\dot{x} - k\,x^2$

13. Generalized co-ordinate can be of any dimension. However

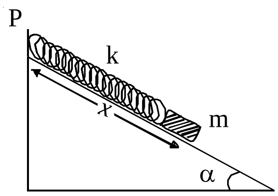
(A) the product of generalized co-ordinate and generalized momentum is always of dimension $M\,L^2\,T^{-1}$.

(B) dimension of the work done depends on the dimension of the generalized force.

(C) the generalized momentum is always of the dimension $M\,L\,T^{-1}$.

(D) the generalized force is always of the dimension $M\,L^2\,T^{-2}$.

14. A body of mass m is sliding down a frictionless inclined plane of angle α with the horizontal. A massless spring of stiffness constant k and equilibrium length l attaches the body to the apex P of the plane, as shown in the figure. If the distance of the body from the apex P of the plane is x, a possible Lagrangian of the system will be



(A) $L = \frac{1}{2}\,m\dot{x}^2 - mg\,\cos\,\alpha - kx$

(B) $L = \,m\dot{x}^2 - 2mg\,\cos\,\alpha + 2kx$

(C) $L = \,m\dot{x}^2 + 2mg\,\cos\,\alpha - 2kx$

(D) $L = \frac{1}{2}\,m\dot{x}^2 + mg\,\sin\,\alpha + kx$

15. The Lagrangian for a system of three particles is $L = \frac{1}{2}\left(\dot{q}_1^2 + \dot{q}_2^2 + \dot{q}_3^2\right) - \omega^2\left(q_1^2 + q_2^2 + q_3^2 - q_1\,q_3\right)$. One of the eigenfrequencies is given by

(A) ω

(B) $\frac{\omega}{\sqrt{2}}$

(C) $\sqrt{2}\,\omega$

(D) $\left(2\omega^2\right)^{1/4}$

16. A particle of nest mass m_0 has an energy $2m_0c^2$. It is moving with a velocity of

(A) $\frac{c}{2}$

(B) $\frac{c}{\sqrt{2}}$

(C) $\frac{c}{\sqrt{3}}$

(D) $\frac{\sqrt{3}c}{2}$

17. A small bar magnet falls under gravity through a horizontal loop of conducting wire. The induced *e.m.f.* in the wire

(A) will oscillate rapidly, averaging out to zero.

(B) will increase to a maximum and then decrease, maintaining the same direction.

(C) will increase to a maximum, flip direction and then decrease.

(D) will increase sharply as the magnet begins to fall and then decrease monotonically.

18. A possible vector potential $\vec{A}(x, y, z)$ corresponding to a constant magnetic field $2B$ along the positive *y-axis* is

(A) $\vec{A} = -2B(x\hat{i} - z\hat{k})$

(B) $\vec{A} = B(x\hat{i} - y\hat{k})$

(C) $\vec{A} = -B(z\hat{i} - x\hat{k})$

(D) $\vec{A} = 2B(y\hat{i} - x\hat{k})$

19. A proton P and an alpha particle α undergo a collision and recoil with equal (nonrelativistic) momenta in opposite directions. The ratio of the electromagnetic power radiated by them, i.e., P_α / P_p is approximately

(A) 4

(B) $\frac{1}{16}$

(C) $\frac{1}{4}$

(D) $\frac{1}{2}$

20. An unpolarized light falls on a glass plate at an angle of incidence equal to Brewster's angle. The reflected ray is

(A) linearly polarized perpendicular to the plane of incidence.

(B) linearly polarized parallel to the plane of incidence.

(C) circularly polarized perpendicular to the plane of incidence.

(D) circularly polarized parallel to the plane of incidence.

21. A monochromatic plane electromagnetic wave in free space with amplitude of electric field $1V/m$ is normally incident on a fully reflecting mirror. The pressure on the mirror is (with $\epsilon_0 = 9 \times 10^{-12}$ F/m)

(A) $4{\cdot}5 \times 10^{-12}$ N/m^2

(B) 6×10^{-12} N/m^2

(C) 9×10^{-12} N/m^2

(D) 18×10^{-12} N/m^2

22. Electric field in a region due to a static charge distribution is $\vec{E} = \alpha r^3 \hat{r}$, where α is some constant. Total charge contained in a sphere of radius R around the origin is proportional to

(A) R^3

(B) R^4

(C) R^5

(D) R^6

23. A point charge is at rest close to a grounded spherical metal shell, but without touching it. Electric field due to this point charge at a very large distance r from the charge varies as

(A) $\frac{1}{r}$

(B) $\frac{1}{r^{1{\cdot}5}}$

(C) $\frac{1}{r^2}$

(D) $\frac{1}{r^3}$

24. The strength of magnetic field at the centre of a regular hexagon of side *a* carrying a current *I* is

(A) $\frac{3}{2} \frac{\mu_0}{\pi} \frac{I}{a}$

(B) $3 \frac{\mu_0}{\pi} \frac{I}{a}$

(C) $\sqrt{3} \frac{\mu_0}{\pi} \frac{I}{a}$

(D) $\sqrt{3} \frac{\mu_0}{2\pi} \frac{I}{a}$

25. Let the electric field produced by the charge q_i be given by $\vec{E}_i$. The electric field produced by $\sum_i q_i$ is given by the vector sum $\sum_i \vec{E}_i$. This happens because

(A) Maxwell's equations are Lorentz invariant.

(B) electric field is a gauge invariant quantity.

(C) total charge is conserved.

(D) Maxwell's equations are linear differential equations.

26. A particle of mass *m* and charge *q* is released from a distance *d* from an infinite grounded conducting plane with initial velocity zero. If the gravitational force on the particle can be neglected than its velocity, when it is at a distance $\frac{d}{2}$ from the plane, is given by

(A) $v = \frac{1}{d}\sqrt{\frac{q^2}{8\pi \in_0 m}}$

(B) $v = \sqrt{\frac{q^2}{8\pi \in_0 d}}$

(C) $v = \frac{q^2}{4\pi \in_0 m\sqrt{d}}$

(D) $v = \frac{q}{4\pi \in_0 md}$

27. The state of polarisation of light with the electric field vector $\vec{E} = E_0 \cos(kz - \omega t)\left[\hat{i} - \hat{j}\right]$ is

(A) linearly polarized along z-direction.

(B) circularly polarized.

(C) elliptically polarized with the major axis along *x*-axis.

(D) linearly polarized at – 45° to *x*-axis.

28. In inorganic scintillation detectors, one uses Sodium Iodide and not Sodium Chloride because

(A) NaCl is hygroscopic but NaI is not.

(B) Atomic number of Iodine is higher than that of Chlorine.

(C) Chlorine is more corrosive than Iodine.

(D) NaI has more closely packed crystal structure than that of NaCl.

29. Three defective coins, for each of which the probability of a head turning up is 60%, are tossed simultaneously. What is the probability that there will be two heads and a tail?

(A) 1·6

(B) 0·450

(C) 0·432

(D) 0·144

30. In atomic absorption spectroscopy, which of the following is the generally used radiation source?

(A) Tungsten lamp

(B) Xenon mercury arc lamp

(C) Hollow cathode lamp

(D) Hydrogen or deuterium discharge lamp

31. Which of the following functions cannot be fitted with experimental data by linear least square method even after variable transformation?

(A) $y = a \sin(bx)$

(B) $y = \frac{a}{x^2} + bx$

(C) $y = ae^{-bx}$

(D) $y = a \sinh x$

32. If the output of the system is $y(n) = \sum_{k=-\infty}^{n} x(k)$ with an input of $x(n)$, then the system will work as

(A) an accumulator

(B) an adder

(C) a subtractor

(D) a multiplier

33. The wave function for a quantum particle is written as

$\psi(x, t) = \psi_1(x) \exp(i E_1 t / \hbar) + \psi_2(x) \exp(i E_2 t / \hbar)$, where $\psi_1(x)$ and $\psi_2(x)$ are real, and are solutions of the time-independent Schrödinger equation with energy eigenvalues E_1 and E_2 respectively. $\psi(x, t)$ represents

(A) a stationary state.

(B) a non-stationary state.

(C) an eigenstate of the same Hamiltonian with eigenvalue $E_1 + E_2$.

(D) a state in which the expectation value of momentum is zero.

34. For three non-identical spin half particles what are the possible eigenvalues of the operator $(\vec{S}_1 + \vec{S}_2) \cdot \vec{S}_3$ (in the unit of $\hbar^2$)

(A) $-1, \ 0, \ \frac{1}{2}$

(B) $-1, \ 0, \ +1$

(C) $-\frac{1}{2}, \ 0, \ +\frac{1}{2}$

(D) $0, \ 1, \ \frac{3}{2}$

35. An electron is in the state $\psi_{3,\,2,\,-2}$ of the hydrogen atom. The probability current for this electron is

(A) identically zero.

(B) directed radially inwards.

(C) a circulation round the nucleus.

(D) a sinusoidal variation with the azimuthal angle φ.

36. A fine platinum needle is brought close to a metal surface in vacuum and a potential is supplied between the metal and the needle, so that electrons can tunnel across the gap between the needle and the metal surface. If *x* be the separation between the tip of the needle and the nearest point on the surface, the current *I* in the needle can be written in terms of two positive constants I_0 and λ as

(A) $I = I_0 - \lambda x$

(B) $I = I_0 \cos \lambda x$

(C) $\log I = I_0 + \lambda x$

(D) $\log I = I_0 - \lambda x$

37. A particle is confined in a one-dimensional infinite square well of length L. The expectation value of the momentum operator with respect to the ground state wavefunction is

(A) 0

(B) $\frac{\hbar}{2L}$

(C) $\frac{\hbar}{L}$

(D) $\frac{\pi\hbar}{L}$

38. If $\hat{L}_x, \hat{L}_y$ *and* $\hat{L}_z$ represent the angular momentum operators, then the commutator $\left[\hat{L}_x, \left[\hat{L}_x, \hat{L}_y\right]\right]$ evaluates to

(A) $\hbar^2 \hat{L}_y$

(B) $\hbar^2 \hat{L}_z$

(C) $-\hbar^2 \hat{L}_x^2 \hat{L}_y$

(D) zero

39. A gas of free electrons is confined in a cubical box of side 100 *nm*. The escape velocity of these electrons may be estimated as around

(A) $1\, m s^{-1}$

(B) $1\, mm s^{-1}$

(C) $10\, m s^{-1}$

(D) $1\, km s^{-1}$

40. A one-dimensional harmonic oscillator of mass m and ground state energy E_0 is subjected to a perturbation $\lambda p^2 = H'$, where p is the momentum and λ is a small positive quantity. The increase in the ground state energy is

(A) $\frac{-\hbar m E_0}{\lambda}$

(B) $\frac{2\hbar \lambda E_0}{m}$

(C) $-2\lambda m E_0$

(D) $\lambda m E_0$

41. The different terms in the Born-Neumann series correspond to

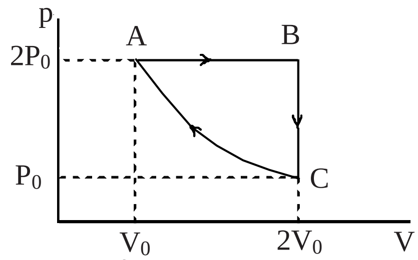
(A) multiple expansion of the potential.

(B) multiple scattering inside the target.

(C) energy levels of bound states in the target.

(D) successive angular momentum states in the incident beam.

42. An ideal gas is subjected to the cycle ABCA shown in the *pV* diagram below:



where the change from C to A is isothermal. The work done by the gas in this cycle is

(A) $(1-\ln 2)P_0 V$

(B) $2\ln 2 P_0 V_0$

(C) $2(1-\ln 2) P_0 V_0$

(D) $4 P_0 V_0$

43. Number of photons radiated per unit time per unit area of a blackbody at temperature T is proportional to

(A) T

(B) T^2

(C) T^3

(D) T^4

44. For a two-dimensional photon gas of energy E, the density of states is proportional to

(A) $\sqrt{E}$

(B) $E^{3/2}$

(C) E^2

(D) E

45. It is necessary to apply quantum statistics to a system of particles if

(A) there is substantial overlap between the wavefunctions of the particles.

(B) the mean free path of the particles is comparable to the inter-particle separation.

(C) the particles have identical mass and charge.

(D) the particles are interacting with each other.

46. A gas mixture consists of 2 moles of oxygen and 4 moles of argon at temperature T. Neglecting all vibrational modes the total internal energy of the system is

(A) 15 RT

(B) 11 RT

(C) 9 RT

(D) 7 RT

47. For an adiabatic process of an ideal monatomic gas, consisting of molecules of mass m, the speed of sound is given by

(A) $\sqrt{\frac{5\, k_B T}{4m}}$

(B) $\sqrt{\frac{3m\, k_B T}{4}}$

(C) $\sqrt{\frac{5\, k_B T}{3m}}$

(D) $\sqrt{\frac{4m}{3\, k_B T}}$

48. In a system of N photons, half of them are polarised in the horizontal direction and the rest in vertical direction. The entropy of the system for $N >> 1$ is

(A) 0

(B) $k_B N \ln N$

(C) $\frac{k_B N}{2} \ln 2$

(D) $k_B N \ln 2$

49. If a process is isobaric then its infinitesimal change of enthalpy ΔH is

(A) zero.

(B) equal to the work done by the system.

(C) equal to the change in internal energy ΔU of the system.

(D) equal to the heat absorbed by the system.

50. Consider a linear simple harmonic oscillator of mass m and frequency ν. The number of microstates in the energy range E and $E + \delta E$ is given as

(A) $\frac{1}{h\nu}\sqrt{E}\ \delta E$

(B) $\frac{1}{h\nu} E\ \delta E$

(C) $\frac{1}{h\nu}\ \delta E$

(D) $\frac{1}{h\nu}\ E^{3/2}\ \delta E$

ROUGH WORK